Galileo as Physicist and Polemicist: A Commentary on an Unpublished Mid-Twentieth-Century Pedagogical Essay

Galilée en tant que physicien et polémiste : commentaire sur un essai pédagogique inédit du milieu du XXe siècle

Ishaan S. Goswami[1*]

1. University of Ottawa, Ottawa, ON, Canada
*Corresponding author. Email: igosw085@uottawa.ca

Abstract | Résumé

This commentary examines an unpublished mid-20th-century pedagogical essay on Galileo Galilei authored by physicist and educator Mam Chand Jain. Written within a classical physics teaching tradition that integrated history, philosophy, and mathematical derivation, the essay presents Galileo not merely as a source of foundational laws but as a persuasive figure whose scientific arguments effected a broader worldview shift. The original work combines biographical reflection, mechanical analysis of Galileo's contributions to classical mechanics and astronomy, and a mathematical–philosophical treatment of the so-called Galileo–Plato problem concerning the "abode of God."

In this commentary, the essay is situated within its historical and educational context, highlighting both its conceptual strengths and its limitations in light of subsequent historiography and physical theory. Particular attention is given to its pedagogical method, which foregrounds the development of physical intuition through historical argumentation rather than rote formalism. This commentary argues that such integrative approaches remain valuable for contemporary scientific pedagogy, offering an integrated framework for understanding how scientific knowledge is discovered, communicated, and taught.

Ce commentaire examine un essai pédagogique inédit du milieu du XXe siècle portant sur Galileo Galilei, rédigé par le physicien et éducateur Mam Chand Jain. Rédigé dans une tradition d'enseignement de la physique classique intégrant l'histoire, la philosophie et la dérivation mathématique, cet essai présente Galilée non seulement comme une source de lois fondamentales, mais aussi comme une figure persuasive dont les arguments scientifiques ont provoqué un changement plus large de vision du monde. Le texte original combine une réflexion biographique, une analyse mécanique des contributions de Galilée à la mécanique classique et à l'astronomie, ainsi qu'un traitement mathématique et philosophique du problème dit de Galilée–Platon concernant la « demeure de Dieu ».

Dans ce commentaire, l'essai est replacé dans son contexte historique et éducatif, en mettant en évidence à la fois ses forces conceptuelles et ses limites à la lumière de l'historiographie et des théories physiques ultérieures. Une attention particulière est accordée à sa méthode pédagogique, qui met en avant le développement de l'intuition physique à travers l'argumentation historique plutôt que par un formalisme répétitif. Ce commentaire soutient que de telles approches intégratives demeurent pertinentes pour la pédagogie scientifique contemporaine, en offrant un cadre unifié pour comprendre comment le savoir scientifique est découvert, communiqué et enseigné.

Keywords: Galileo Galilei; History and Philosophy of Science; Physics Pedagogy; Classical Mechanics; Scientific Revolution; Scientific Communication

Introduction

This article presents a commentary on and transcription of an unpublished pedagogical essay (See Appendix A) on Galileo Galilei written by Mam Chand Jain, a physicist and educator trained in the mid-20th century in both the Indian and British academic traditions. The manuscript was begun in the 1950s and formally typed in 1972. It was intended as a pedagogical synthesis combining a first-principles exposition of Galileo's discoveries in classical mechanics with historical analysis of the Scientific Revolution. Jain's paper serves as a biographical portrait of Galileo

Figure 1. Mam Chand Jain (1923–2014), physicist and educator. Photograph taken during his teaching career in Leader, Saskatchewan (c. 1966). Author of the 1972 paper "Galileo."

as a personality; a technical exposition of Galileo's physics and astronomy; and a mathematical and philosophical analysis of the Galileo-Plato problem - the "place of God" problem. This paper reflects a period when physics education emphasized conceptual foundations, history, and philosophy, rather than highly formalized problem-set-driven instruction. Galileo is not portrayed as the founder of several axioms of classical physics, but rather as a pedagogical example in the history of science. The essay reproduced here is a record of how science was once taught as a unified intellectual enterprise.

Beyond its pedagogical value, the essay contains a detailed mathematical analysis of Galileo's speculative claim regarding a single "abode of God" from which planets were set into motion. Using Keplerian relations and later Newtonian reasoning, Jain demonstrates that Galileo's proposal is internally inconsistent and systematically examines successive reformulations of the problem by later thinkers. This analysis does not refute Galileo's reasoning within his historical context, nor does it challenge the legitimacy of his speculative premise. Rather, it examines the assumptions underlying his proposal using physical laws and relations developed after his time.

Mam Chand Jain (1923–2014), B.Sc., M.Sc., P.G.C.E., was trained in physics and mathematics, earning a bachelor's degree in chemistry, physics, and mathematics and a master's degree in physics with a focus on astronomy and spectroscopy, followed by postgraduate teaching qualifications from the Institute of Education, University of London. He spent over three decades teaching physics and mathematics at the secondary and post-secondary levels in India, Ethiopia, Ghana, the United Kingdom, and Canada. His academic formation emphasized classical mechanics, mathematical derivation, and the historical development of physical theory, reflecting a pedagogical tradition in which history and philosophy of science were integral to physics instruction. This pedagogical orientation is reflected in one of the texts Jain used both as a student and later as a source for his essay: The Pre-University and Intermediate Physics by Basu and Chatterji (2), in which dense mathematical derivations are interwoven with historical context and epistemic reflection

Conceptual Strengths of the Essay

Galileo was a persuader, not just an experimenter

Before the notion that science communication was a noteworthy pursuit, Galileo was a polemicist who fought for his interpretation of the physical universe, as opposed to previous Aristotelian notions of cosmology. Galileo was not afraid of confrontation, and rather, was willing to engage in fierce debates with his opponents, ultimately leading to his trial and condemnation by the Catholic Church. Galileo understood that science had to advance socially, not just logically, as his arguments were opposed to the neo-Platonic and Aristotelian cosmology accepted by the church. Long before 'science communication' became a formal discipline, this essay recognizes that Galileo's success depended as much on persuasion as on proof.

Physics as worldview shift

The essay understands Galileo as dismantling Aristotelian cosmology, not merely adding new observational data. While heliocentrism is now accepted as a physical fact, in Galileo's era it represented a profound epistemic and existential rupture. The displacement of the Earth from the center of the cosmos challenged deeply held assumptions about order, purpose, and humanity's place in creation. The emotional and intellectual shock of heliocentrism is thus foregrounded as a necessary component of scientific transformation, not a peripheral consequence.

Serious engagement with mechanics

Jain explicitly outlined the mechanical derivation of Galileo's observations of projectile motion, inertia, inclined plane reasoning, and refinements under Newtonian dynamics. This paper was structured as such to situate the mathematics and physics in the world in which it was discovered. The mechanics are presented as arguments constructed in response to physical phenomena – not laws divorced from the reality they were meant to describe.

Historical and Scientific Limitations

The essay necessarily reflects the historiographical and scientific context of its time. Subsequent scholarship has questioned the historicity of the Tower of Pisa experiment (3). As well, this paper does not address Galileo's findings in the context of later developments in thermodynamics and relativity. However, for a physicist trained in the classical tradition, these limitations can be understood as products of historical circumstance. Read as a pedagogical document rather than a contemporary research contribution, the essay remains internally coherent and intellectually rigorous within its intended framework.

Pedagogical Significance Today

Modern science education is increasingly fragmented, with students learning techniques before understanding meaning, and how the axioms and theorems they assume to be true were discovered. This essay models an integrated approach, wherein history, philosophy, mathematics, and experimentation are all acknowledged in order to provide a deeper intuition behind these discoveries. Revisiting such pedagogical texts is a reminder that science is not merely a body of results, but a way of thinking about nature, evidence, and truth.

The essay exemplifies an educational philosophy in which the history and philosophy of science functioned as tools for developing physical intuition. By tracing Galileo's arguments, errors, and rhetorical strategies, students were encouraged to understand why modern mechanics emerged, not merely how to apply its equations.

Editorial Conflict of Interest Statement

The author of the original Galileo essay, Mam Chand Jain, was a physicist and educator and the maternal grandfather of the a author Ishaan S. Goswami. This commentary was handled independently by the OSURJ editorial team, and the author was not involved in the review or acceptance decision. Publication of the original essay and related archival materials was undertaken with the permission of the estate of Mam Chand Jain.

References

1. M. C. Jain, Galileo (unpublished manuscript, 1972).
2. N. Basu, J. Chatterjee, The Pre-University and Intermediate Physics (H. Chatterji & Co., Calcutta, 1954). Available from: https://archive.org/details/dli.ernet.240843
3. M. Segre, Galileo, Viviani and the tower of Pisa. Stud. Hist. Philos. Sci. A 20, 435–451 (1989).

Appendix A: Transcribed Text of “Galileo” (1972)

This appendix presents a faithful transcription of Mam Chand Jain’s unpublished 1972 pedagogical essay “Galileo.” The text has been re-typed for clarity and accessibility. Spelling, punctuation, and mathematical notation have been preserved wherever possible. No substantive content has been altered. The manuscript reflects the conventions of physics pedagogy and historical interpretation of its time. Mathematical notation, terminology, and historiographical claims are preserved as originally presented. The text is provided as a historical document and should be read within its pedagogical and temporal context.

Galileo
August 4, 1972
M. C. Jain

Today, I will honor a great man who was born just four centuries ago, a man whose achievements have touched on almost all fields of human effort. Galileo was a powerful, passionate figure, a man who dominated every room and every discussion he entered. His excitement over the new world he saw opening up, and his blistering intolerance of those who would not see it as he did, break through in every page of his writings. As we read his letters we can hardly help falling into step behind his banners, we laugh with him, lock swords with his enemies, rejoice at his triumphs. His is no clean cut world of concepts and theorems, but a brawling world of clashes and schemes, no place for a scientist, I would say. But a sort of place in which someone who has set himself to tearing down an ageold system of thought and replacing it with a new is likely to feel at home.

Galileo loved the fray. Not for him the laborious hours of observation of a Tycho; not for him the endless calculations and curve fittings of a Kepler. He was a man with a vision of the way, the universe had to be, and a talent for communicating that vision to others. None knew better than Galileo that, whereas theorems have to be proved, people had to be persuaded. To someone as strongly convinced of his own righteousness as Galileo was proof is at best secondary. But when someone has a message as novel and as far reaching as Galileo had, the ability to persuade others of its worth is altogether vital. Most of his professional life was spent, not in observing, not in calculating, not in proving, but simply in persuading. He had to convince reluctant hearers that what he had to say about Nature made far more sense than anything that had ever been said before. His historic role was to change a world view, and this demanded talents of a far more diverse order than would be required by a simple establishment of a new theory. His works are thus characterized by a vigor and an immediacy that set them apart in the annals of scientific writing. They are exuberant, brash, speculative and wheedling by turn. His way of deployment of scientific method turned out almost immediately to have a power that set off the new science sharply from the old.

Galileo was the man who was a legend almost in his own life time and who has never since ceased to light up man's imagination. His life, his work, his death were all of a piece; there was the sort of symbolic unity about his career that sets up an immediate resonance in anyone who shares even a part of the vision that animated it. In Galileo, I see a vast energy directed single-mindedly to the changing of history.

Galileo's work marked the watershed between old and new. Galileo is portrayed as the pioneer of a new spirit, the father of the new sciences. Galileo's science had to be conceived as a violent creative break with all that had gone before.

Galileo published the great Discourse that contains the germ of Newtonian Mechanics both in its results and its methods, this mode of approach suggests that the transition from medieval to modern science was accomplished almost single-handedly by Galileo.

Galileo was born in Pisa in 1564, within a year of Shakespeare's birth. When Galileo was 11 years old he was sent to school in a monastery. At the age of 15, his father sent him to study medicine under a famous doctor Cesalpino but medicine was not Galileo's best love. At 19 years of age, he left university without taking examination and managed to study algebra from Professor Ostilio Ricci at Florence. He discovered the beauty of Mathematics and thereafter devoted his life to it, Physics, and Astronomy. Not only was he a great mathematician but also an excellent musician in both performance and composition. Galileo was an adept experimenter. As a boy he was fond of making mechanical toys. The training in Manual dexterity, which he thus acquired, was invaluable asset in all his scientific work.

He was also brilliant student, his intellectual curiosity was avid, his perception quick and his retention tenacious.

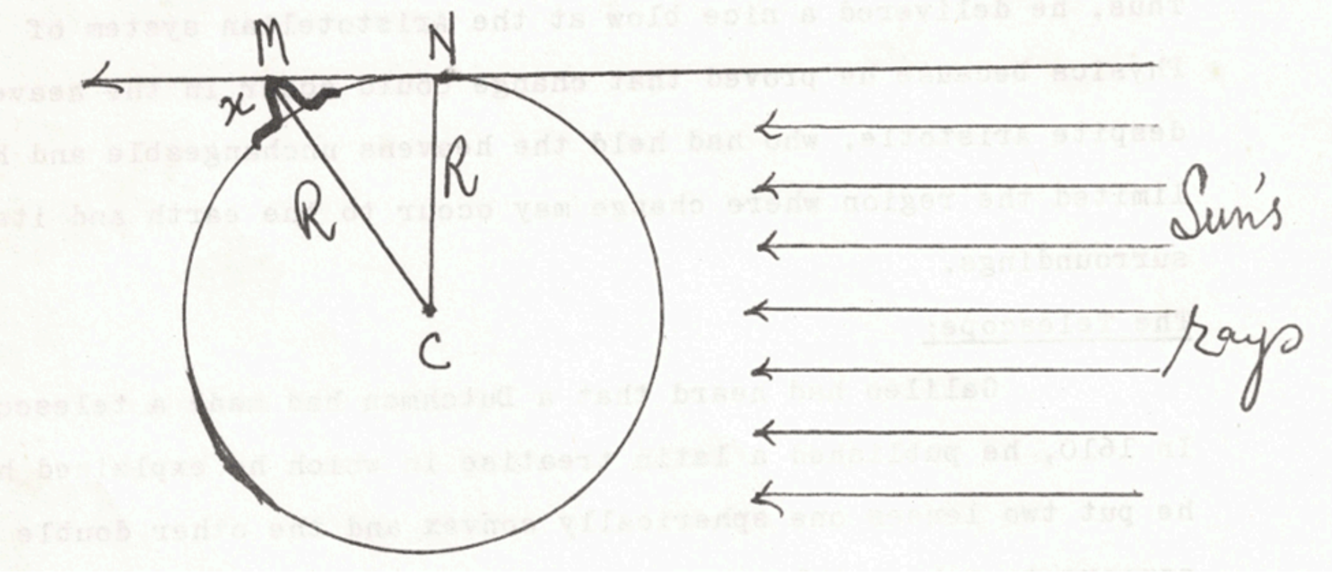
During his studies as a medical student he made his first invention, the pulsilogium for reading the pulse of a patient. The pulsilogium was essentially a pendulum of adjustable length. The doctor had to synchronize the pulse beat with the oscillations of the pulsilogium and read the post directly from a scale. He was a genius in the selection of a problem and method of attack.

Galileo made his first contribution to Astronomy before he ever used a telescope. In 1604, Galileo showed a nova to be a true star located out in the celestial spaces and not inside the sphere of the moon. Galileo showed that this new star had no measurable parallax and so was very far from the earth. Thus, he delivered a nice blow at the Aristotelian system of Physics because he proved that change could occur in the heavens despite Aristotle, who had held the heavens unchangeable and had limited the region where change may occur to the earth and its surroundings.

<u>The Telescope:</u>

Galileo had heard that a Dutchman had made a telescope. In 1610, he published a Latin treatise in which he explained how he put two lenses one spherically convex and the other double concave to make a telescope of his own. Objects when looked through his third telescope appeared magnified almost a thousand-fold in area and thirty times nearer. With the help of this telescope he made astonishing discoveries of sun spots, the mountains and craters on the surface of the moon. He found that the surface of the moon is not smooth, uniform and precisely spherical as a great number of philosophers believed it to be, but it is uneven, rough and full of cavities and prominences, being not unlike the face of the earth, relieved by chains of mountains and deep valleys. Not only did Galileo describe the appearance of mountains on the moon but also measured them. Galileo's determination of the height of the mountains on the moon has withstood the test of time and even today we agree with his estimate of their maximum height

Galileo’s Measurements of the Height of Mountains on moon:



N = the terminator (boundary) between the illuminated and non-illuminated positions of the moon.
M = bright spot observed in the shadowed region.

$$(R + x)^2 = R^2 + MN^2 \text{ (Pythagorean Theorem)}$$

$$R^2 + 2Rx + x^2 = R^2 + MN^2$$

$$x^2 + 2Rx - MN^2 = 0$$

Solved for x, the altitude of the mountain.

Radius of the moon was calculated from the distance of the moon from the earth which was known to Galileo.

Galileo also discovered the phases of Venus like those of our moon and four planets of Jupiter. Jupiter is now known to have at least 12 moons. He found that the milky way consisted of a great number of individual stars. He also saw the morning star.

In 1610, he left Padua and took a position as court philosopher to the Duke of Tuscany. In 1611, he discovered the handles of Saturn. His telescope was not powerful enough to define the phenomena as a ring. The same year, he saw spots on the Sun and realized that planets rotated. Copernicus had described planets rotated but for the first time a man had seen rotations of the planets.

Galileo also deduced, from study of the moon in its various phases that the earth turned about its axis and revolved around the sun as Copernicus had said and that it reflected light as the other planets. He saw the phases of Venus and struck great blow against the old astronomy. If Venus did travel in epicycles as held before that she would have no quarter, half and full phases as seen by Galileo.

Prior to 1609, the Copernican system of the universe had seemed to men a mere mathematical speculation. The basic supposition that the earth was merely another planet had been so contrary to all the dictates of experience and common sense that very few men had faced up to the awesome consequences of the heliostatic system. But after 1609, when men discovered through Galileo’s eyes what the universe was like, they had to accept the fact that the telescope showed the world to be non-Ptolemaic, non-Aristotelian, in that the uniqueness attributed to the earth could not fit the facts. Thus, gone forever was the concept that the earth had a fixed spot in the centre of the universe, but it was now conceived to be in motion. Gone also was the comforting thought that the earth is unique, that it is an individual object without any likeness anywhere in the universe, that the uniqueness of man had given a uniqueness to his habitation.

Galileo’s Other Works:

In 1597, he invented a compass for the direction of long distance. He elaborated and wrote about his theories of motion. He experimented in ballistics and the velocity of projectiles. He studied and described the strengths of materials, vital to construction of buildings and machinery. Galileo has been called father of dynamics. He defined various types of motion. Amongst others the definitions of uniform motion and uniform acceleration are of particular interest. The following paragraph is a description in his own words:

> “I consider a motion steady or uniform if the distances traversed by the moving body during any equal time intervals are equal……. I say that motion is steadily or uniformly accelerated which acquires, in any equal time equal increments of velocity.”

Aristotle believed that the bodies fall freely with speeds that are proportional to their weights, but Galileo proved it to be wrong by allowing a card board placed over a coin with no edges projecting outwards that both gained speed at the same rate. Thus, he introduced the idea of air friction or friction of the medium.

In this connection, a historical passage from his book, “The Dialogue Concerning the Two New Sciences” is given… the book is presented in the form of a dialogue between three persons, Salviati, the teacher and a man on knowledge is the mouthpiece of Galileo. Salgredo is the neutral who asks intelligent questions. The character Simplico, Galileo claims that he derived from the name Simplicius, a student of Aristotle in the sixth century.

He performed the greatest historical experiment and showed the world that two pieces of iron of different weights allowed to fall together from the top of the leaning tower of Pisa touch the ground together.

Galileo’s biographer, Viviani, who knew Galileo during his last years told a fascinating story which had taken root in the Galileo legend. According to Viviani, Galileo, desiring to confute Aristotle, ascended the Leaning Tower of Pisa, “in the presence of all other teachers, philosophers and all students and by repeated experiments proved that the velocity of moving bodies of the same composition, unequal in eight, moving through the same medium, do no attain the proportion of their weight as Aristotle assigned it to them, but rather they move with equal velocity.

He, with the help of a pail of water, determined time up to 1/10th of a second.

In order to find how did the velocity change with time, he had to modify his experiment and let a ball roll down a sloping plank as he said to dilute the gravity i.e. to slow the motion or in other words decrease the acceleration.

With the aid of these experiments, Galileo demonstrated that:

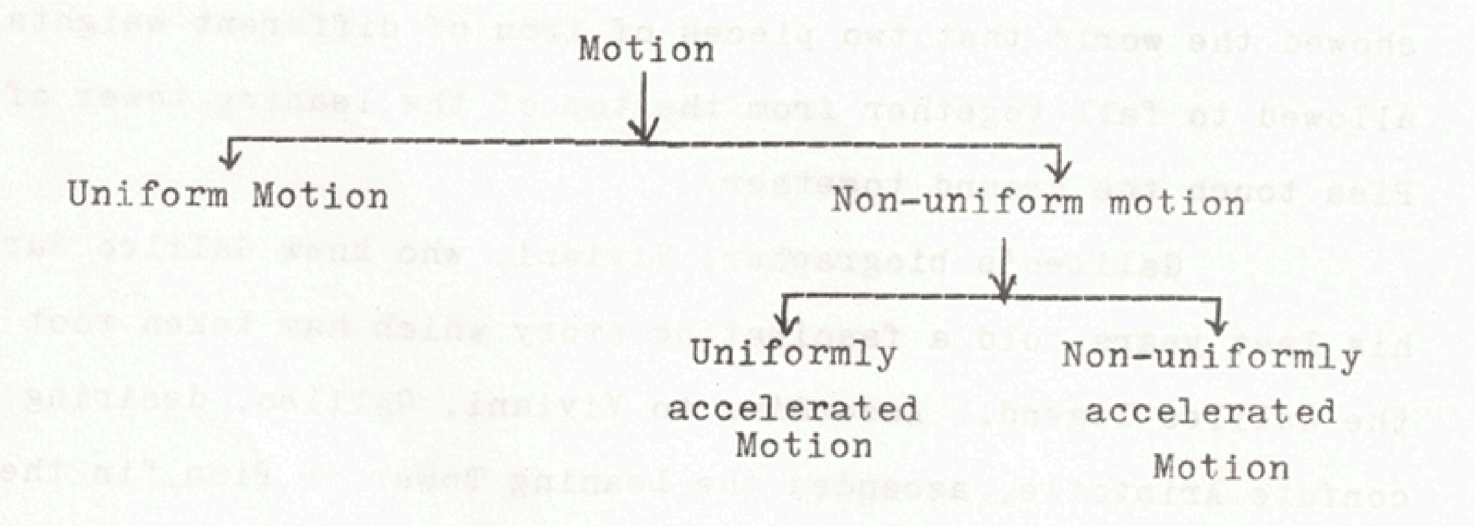
$$v \propto t$$

$$s \propto t^2$$

Where
v = speed,
t = time,
s = distance

For any inclination of the plane, however steep.

Galileo presented the following scheme to correct the Aristotelian Law of Motion



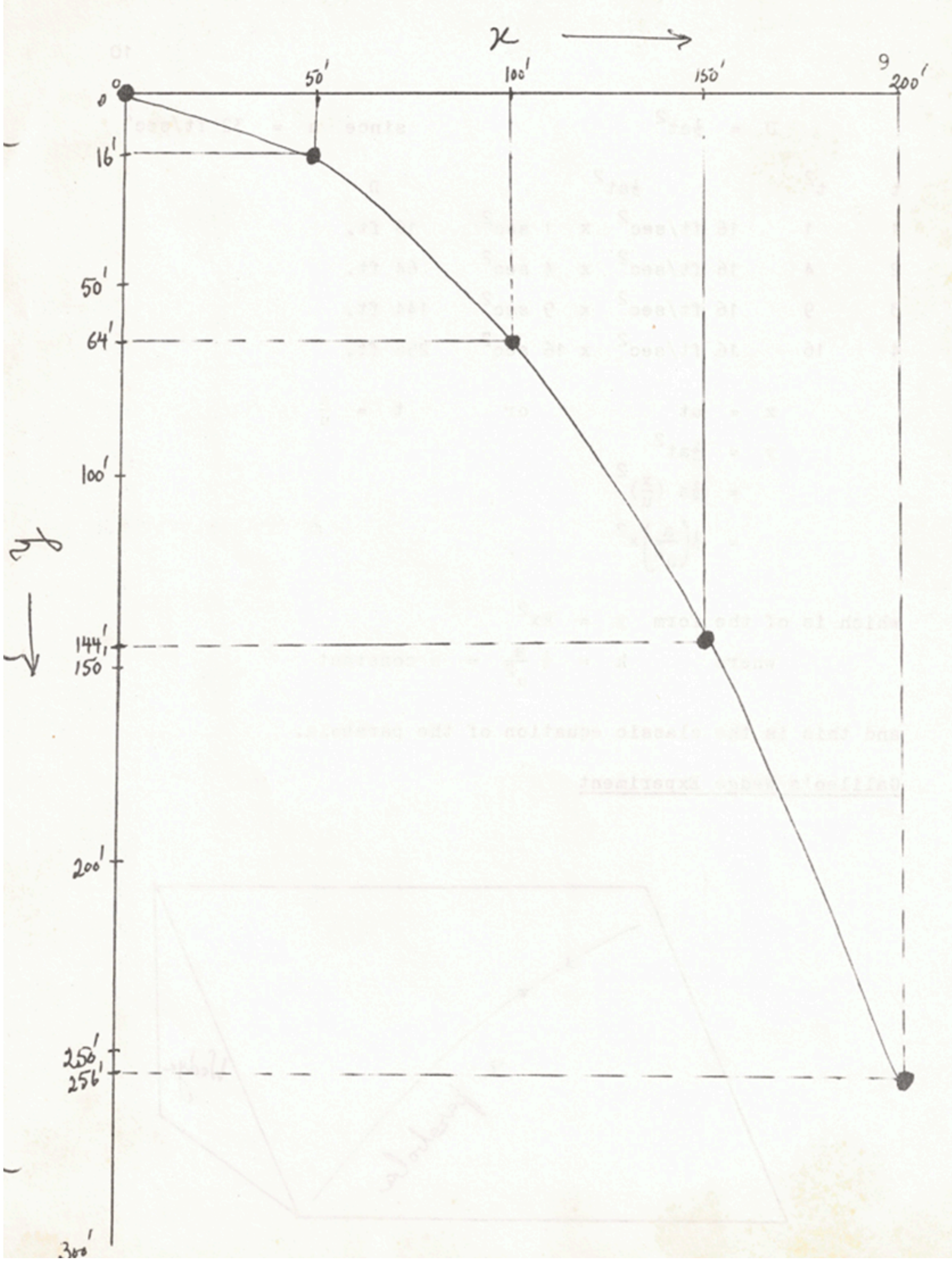
Galileo applied the fourteenth century “mean speed rule” to his Experiment of a “free fall of a body” and showed that”

$$s = \left(\frac{v_1 + v_2}{2}\right) t$$

Galileo also demonstrated that a projectile follows the path of a parabola because the projectile has simultaneously a combination of two independent motion:

(i) A uniform motion in a forward direction
(ii) A uniformly accelerated motion downward

Galileo analyzed the projectile motion considering a shell fired horizontally from a cannon at the edge of a cliff at a speed of 50 ft. per second.



$$D = \frac{1}{2}at^2, since\ a = 32\ ft/sec^2$$

t	t^2	1/2 at^2	D
1	1	16 ft/sec^2 x 1 sec^2	16 ft.
2	4	16 ft/sec^2 x 4 sec^2	64 ft.
3	9	16 ft/sec^2 x 9 sec^2	144 ft.
4	16	16 ft/sec^2 x 16 sec^2	256 ft.

$$x = ut\ or\ t = \frac{x}{u}$$

$$y = \frac{1}{2}at^2$$

$$y = \frac{1}{2}a\left(\frac{x}{u}\right)^2$$

$$y = \frac{1}{2}\left(\frac{a}{u^2}\right)x^2$$

which is of the form

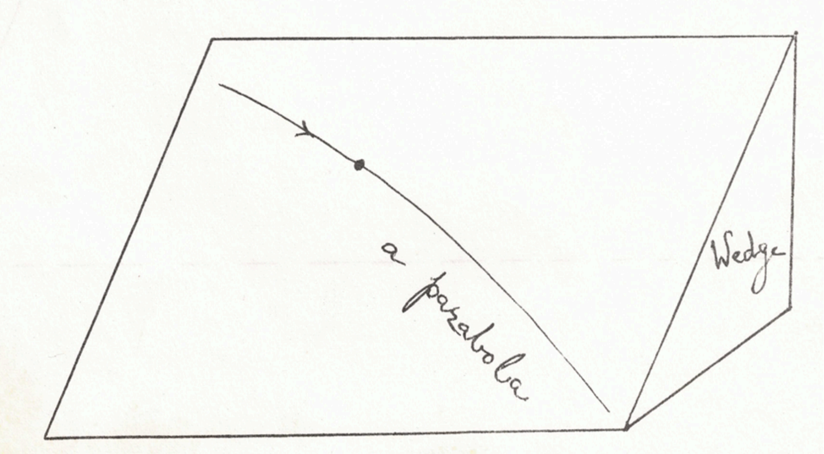
$$y = kx^2$$

where

$$k = \frac{1}{2}\left(\frac{a}{u^2}\right) = a\ constant$$

and this is the classic equation of the parabola.

Galileo’s edge experiment



Inertia:

In one of the experiments, Galileo used two balls of the same size, one of lead and the other of oak, and both were let fall from a height of 200 cubits. Galileo found that balls reached the earth with slight difference in speed. He proved by this experiment that the resistance of the air increases in some proportion to the speed until the resistance of the air equals and offsets the weight, pulling the body down to the earth. Thus, Galileo concluded that when the resistance becomes so great that it equals the weight of the falling body, the resistance of the air will prevent any increase in speed and will render the motion uniform. This is anti-Aristotelian, because Aristotle held that when the motive force equals the resistance, the speed is zero. Galileo's principle is in limited form, a statement of Newton's first law of motion or the principle of inertia. This may be considered one of the major foundations of Modern Newtonian Physics.

In Dialogue, Galileo has hit upon the principle of inertia:

(i) A ball on a sloping plane would accelerate spontaneously
(ii) A ball on an upward slope would need a force to go up or to remain still
(iii) A ball placed upon a surface with no slope upward or downward would remain at rest. But if it is pushed in a direction it will move in that direction. If the surface is unbounded the motion would be boundless or the motion will be perpetual.

Galileo used the term quantity of motion (M) for momentum and 'w' for mass:

$$\therefore M = w \times u$$

He had keen sense of observation and interpretation. He studied vibrations, particularly in churches, of hanging lamps, etc. that the frequency of a simple free pendulum is constant and independent of its length provided the angle of swing is not greater than about 20°. As there were no watches he used his pulse as a watch.

Aristotle had told that water is ten times heavier than air. Galileo made a comparative study of the weights of equal volumes of water and air with a crude but elegant experiment. He did the experiment with utmost patience and care using a single grain of sand at a time as a unit of mass. He verified the results with a modified apparatus.

He obtained that water should be 400 times heavier than air volume to volume. The modern ratio of density of water to density of air = 776. This experiment shows how good he was in his ability to predict and then to prove.

He had such an insight that even when he was blind he could sense a good problem from afar. He saw a lift pump which would work perfectly if the level of water stood above a certain level but below that level it failed to work. Aristotelian explanation for the working of pump was that "nature abhors vacuum." Discussing the pump with his students, Torricelli and Viviani, Galileo remarked in his usual gay tone that Damn Nature's horror of the void, by some mysterious whim seemed to peter-out suddenly at about 18 cubits. He suggested if they investigated the matter they would likely learn something important and useful. The work of Torricelli in the field is well known.

During his last years, he worked on several problems, among them was the construction of a pendulum clock which had a dial graduated in minutes and a hand that was operated by the pendulum but it had to be given a nudge every now and then to keep it swinging. Working on the problem on his deathbed in 1642, to Viviani he said, "quick while there is yet breath" and gasped out the specifications of the new instrument. Many years later, the Dutch scientist, Christian Huygen, completed the invention and took out a patent.

As I already stated, he had keen sense of observation and whatever he observed he recorded it. He once saw Chaldni figures and his statement of the phenomena is as follows:

> "As I was scraping a brass plate with a sharp chisel to remove some spots, I heard the plate emit a clear whistling tone. I noticed a long row of streaks (of small particles) parallel to each other. When the tone was higher, the streaks were closer together."

As we know now that these streaks will be formed at nodal lines.

In 1592, Galileo gave the importance to the study of heat, for he constructed a thermometer which expresses temperature numerically and thereby brought the subject to a quantitative stage. As explained by Viviani, he took a glass bulb about the size of a hen's egg with a tube about 2 spans in length and width of a straw-stem, warmed the glass with his hands, and turned it so that the end of the tube is dipped into a tumbler placed underneath. When the air in the bulb cooled, the water rose more than a span above the surface of the liquid in the tumbler. He also showed if the bulb was cooled, with wine or alcohol, the level of water rose still higher. This tube is called Galileo's tube.

Even when sealed thermometer was constructed for the first time, Galileo' device was considered more accurate. Later on in 1810, the Galileo's tube was modified to make a differential thermometer.

Galileo Traces the place of abode of God, The Creator

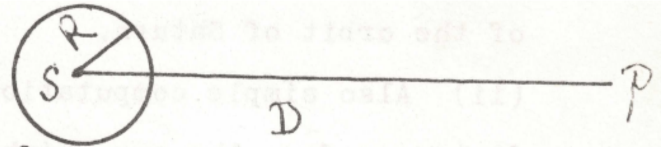
The Galileo-Plato Problem:

In the Dialogo under the guise of a suggestion by Plato, Galileo states that

God made each planet at His place of abode and gave it a straight and accelerated motion so that starting from rest, it gradually attained the particular velocity, He intended to confer upon it. when that velocity had been attained, God converted that straight line motion into a circular motion at the very same speed: a circular motion to be kept perpetually uniform forever after.

Thus, knowing the dimensions of the planetary orbits and the speeds of the planets, the Divine Order may be discovered.

Let R = average distance of a planet from the sun
And T = periodic time of revolution of the planet about the sun
P = place of God
D = distance of P from the sun
a = uniform acceleration of the planets towards the sun



$$V = \frac{2\pi R}{T}$$

$$V^2 = u^2 + 2as$$

$$V^2 = 0 + 2a(D - R)$$

$$\left(\frac{2\pi R}{T}\right)^2 = 2a(D - R)\ (eq\ 1)$$

or

$$\left(\frac{4\pi^2 R^2}{T^2}\right) \times \frac{R}{R} = 2a(D - R)$$

$$4\pi^2 \frac{R^3}{T^2} = 2aR(D - R)$$

$$4\pi^2 K = 2aR(D - R)$$

As K = $\frac{R^3}{T^2}$ (Kepler's Third Law)

$$\frac{4\pi^2 K}{2a} = DR - R^2$$

$$\therefore DR - R^2 = constant$$

$$\therefore DR_1 - DR_1^2 = DR_2 - DR_2^2$$

$$D(R_1 - R_2) = R_1^2 - R_2^2$$

$$D = R_1 + R_2$$

Now R1 for Mercury = 0.4 A.U.
And R2 for Venus = 0.7 A.U

$$\therefore D = 0.4 + 0.7$$

$$= 1.1\ A.U.$$

Analysis of Galileo’s Conception of Heavens:

So God must live a little more than the orbital distance of the earth from the sun.

(i) Which is absurd because by definition, P must lie outside of the orbit of Saturn.

(ii) Also simple computations show that there are no two planetary distances from the sun which can be added together to give the same distance D.

Conclusion:

Thus, there is no single place from which all the planets could be let fall from rest toward the sun with the same uniform acceleration so as to arrive at their respective orbits with speeds of the magnitude of their observed orbital speeds.

Paul Mansion’s Analysis (1894) of Galileo’s Problem:

Galileo’s assumptions

According to equation (1)

$$\left(\frac{2\pi R}{T}\right)^2 = 2a(D - R)$$

$$\frac{4\pi^2 R^2}{T^2 2a} = D - R$$

$$R + \frac{2\pi^2}{a}\frac{R^2}{T^2} = D$$

$$R + m\frac{R^2}{T^2} = D$$

where $m = \frac{2\pi^2}{a}$ = a constant

$$\therefore\ R_1 + m\frac{K}{R_1} = R_2 + m\frac{K}{R_2}$$

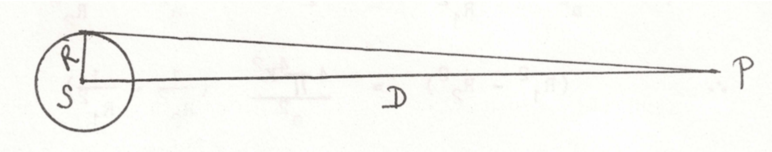
$$R_1 - R_2 = mK\left(\frac{1}{R_1} - \frac{1}{R_2}\right) = mK\left(\frac{R_1 - R_2}{R_1 R_2}\right)$$

$$R_1 R_2 = mK$$

i.e. the product of any pair of sun-planet distances = a constant is plainly absurd.

Mansion considered two further possibilities.

1) All planets do not fall toward the sun but rather that each one drops along a tangent until it meets its proper orbit.



$$D^2 = R^2 + d^2$$

$$d_2 = \frac{1}{2}at^2$$

Where t = time for a planet to fall from P to its orbit

$$V = at$$

$$V = \frac{2\pi R}{T}$$

$$V^2 = a^2t^2$$

$$t^2 = \frac{V^2}{a^2}$$

$$d = \frac{1}{2}at^2$$

$$d = \frac{1}{2}a\frac{V^2}{a^2}$$

$$d = \frac{1}{2}\frac{V^2}{a} = \frac{1}{2}\left(\frac{2\pi R}{T}\right)^2\frac{1}{a}$$

$$d = \frac{2\pi^2R^2}{aT^2}$$

as $D^2 = R^2 + d^2$

$$D^2 = R^2 + \left(\frac{2\pi^2R^2}{aT^2}\right)^2$$

$$D^2 = R^2 + \frac{4\pi^4R^4}{a^2T^4}$$

$$D^2 = R^2 + \frac{4\pi^4}{a^2}K^2\left(\frac{1}{R^2}\right)\ (eq\ 2)$$

For two planets:

$$R_1^2 + \frac{4\pi^4}{a^2}K^2\left(\frac{1}{R_1^2}\right) = R_2^2 + \frac{4\pi^4}{a^2}K^2\left(\frac{1}{R_2^2}\right)$$

$$\therefore (R_1^2 - R_2^2) = \frac{4\pi^4}{a^2}K^2\left(\frac{1}{R_1^2} - \frac{1}{R_2^2}\right)$$

$$(R_1^2 - R_2^2) = \frac{4\pi^4K^2}{a^2}\left(\frac{R_1^2 - R_2^2}{R_1^2R_2^2}\right)$$

$$R_1^2R_2^2 = \frac{4\pi^4K^2}{a^2}$$

$$R_1R_2 = \frac{2\pi^2K}{a}\ \ (eq\ 3)$$

$$= a\ constant$$

There is no constant value obtained by multiplying the pairs of R_1 and R_2.

To continue further
From equation (3)

$$R_2 = \frac{2\pi^2 K}{a}\left(\frac{1}{R_1}\right)$$

But, from equation (2)

$$D^2 = R_1^2 + \frac{4\pi^4}{a^2}K^2\left(\frac{1}{R_1^2}\right)$$

$$\therefore\ D^2 = R_1^2 + R_2^2\ (eq\ 4)$$

By substituting $R_2 = \frac{2\pi^2 K}{a}\left(\frac{1}{R_1}\right)$

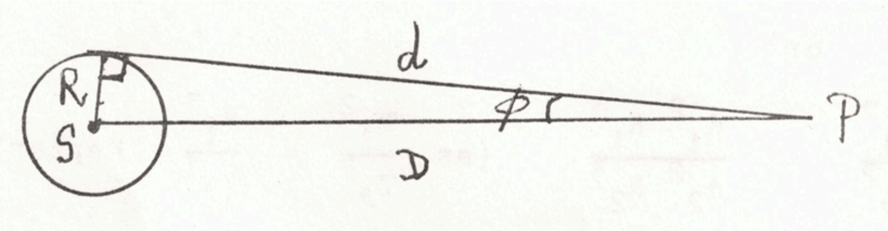
There can be no single value of D for which this equation is valid and hence, no single point from which all the planets may be dropped according to the conditions of the problem.

Finally, suppose each planet falls along a tangent from P to its orbit but its acceleration is not a but component a directed from P to the sun.

Acceleration of the planet:

$$= a\cos\varphi$$

$$= a\frac{d}{D}$$



$$\therefore d = \frac{1}{2}a\cos\varphi\, t^2$$

$$d = \frac{1}{2}a\frac{d}{D}t^2$$

$$\frac{2D}{a} = t^2$$

$$t = \sqrt{\frac{2D}{a}}$$

But

$$V = g\cos\varphi\, t$$

$$V = g\frac{d}{D}t$$

As g, D and t are constant

$$V \propto d$$

Or

$$\frac{V_1}{V_2} = \frac{d_1}{d_2}$$

$$\frac{V_1}{V_2}\frac{T_1}{T_2} = \frac{d_1}{d_2}\frac{T_1}{T_2}$$

$$2\pi R_1 = V_1 T_1$$

$$2\pi R_2 = V_2 T_2$$

$$\therefore \frac{R_1}{R_2} = \frac{V_1}{V_2}\frac{T_1}{T_2} = \frac{d_1}{d_2}\frac{T_1}{T_2}$$

$$\therefore \frac{R_1^2}{R_2^2} = \frac{d_1^2 T_1^2}{d_2^2 T_2^2}$$

$$\frac{R_1^2}{R_2^2} = \frac{d_1^2 R_1^3}{d_2^2 R_2^3}$$

As $\frac{T_1^2}{T_2^2} = \frac{R_1^2}{R_2^2}$ (Kepler's Third Law)

$$\therefore d_1^2 R_1 = d_2^2 R_2 \ (eq\ 5)$$

From the diagram

$$d_1^2 = D^2 - R_1^2$$

$$d_2^2 = D^2 - R_2^2$$

Substituting in equation (5) we get:

$$(D^2 - R_1^2)R_1 = (D^2 - R_2^2)R_2$$

$$\therefore D^2 R_1 - R_1^3 = D^2 R_2 - R_2^3$$

$$D^2(R_1 - R_2) = R_1^3 - R_2^3$$

$$D^2 = \frac{R_1^3 - R_2^3}{R_1 - R_2}$$

$$D^2 = R_1^2 + R_1 R_2 + R_2^2$$

Which is incompatible for the data for any pair of planets.

Newton's Calculations to search for the location of Heavens:

Newton calculated the distance to the Divine Order supposing if a planet is torn off the gravitational force of the sun, how far will it travel before it comes to rest where it was created by God.

Sun's gravitational force on the planet

$$F = ma$$

Where m = mass of the planet, a = centripetal acceleration of the planet

$$F = m\frac{4\pi^2 R}{T^2}$$

$$F = m\frac{4\pi^2 R}{T^2}\frac{R^2}{R^2}$$

$$F = m\frac{4\pi^2}{R^2}\frac{R^3}{T^2}$$

$$F = 4\pi^2 K\frac{m}{T^2}$$

If the planet is made to stop its circular motion and move away from the sun in a straight line to a distance d before coming to rest will gain in potential energy ΔU.

$$4\pi^2 K\frac{m}{R} - 4\pi^2 K\frac{m}{d} = 4\pi^2 Km\left(\frac{1}{R} - \frac{1}{d}\right)$$

But

$$Gain\ in\ U = Loss\ in\ E_k$$

$$\Delta U = \Delta E_k\ (eq\ 6)$$

But

$$\Delta E_k = \frac{1}{2}mV^2$$

$$\Delta E_k = \frac{1}{2}\left(\frac{2\pi R}{T}\right)^2$$

$$\Delta E_k = \frac{2\pi^2 R^2}{T^2}\frac{R}{R}$$

$$\Delta E_k = 2\pi^2 Km\frac{1}{R}\ (eq\ 7)$$

Substitute the values of ΔU and ΔE_k in equation 6 we get:

$$4\pi^2 Km\left(\frac{1}{R} - \frac{1}{d}\right) = 2\pi^2 Km\frac{1}{R}$$

$$2\left(\frac{1}{R} - \frac{1}{d}\right) = \frac{1}{R}$$

$$\frac{2(d - R)}{Rd} = \frac{1}{R}$$

$$2(d - R) = d$$

$$d = 2R$$

If the planet were created at a distance 2R from the sun, the planet would have moved a distance d to be in an orbit round the sun with its present speed such that

$$d = 2R$$

Thus, according to Newton's condition, each planet is dropped from a different point just twice its normal distance from the sun while Galileo observed all planets were let fall from one and the same point.

Next, Newton suggests that if the gravitational power of the sun be diminished by one half.

In this case:

Gain in U as the planet moves away from the sun from the original distance R_1 to a new distance R_2

$$\Delta U = \frac{1}{2}\Delta U_1$$

$$= \frac{1}{2}4\pi^2 Km\left(\frac{1}{R} - \frac{1}{d}\right)$$

$$= 2\pi^2 Km\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$$

$$Gain\ in\ U = Loss\ in\ E_k$$

$$\Delta U = \ \Delta E_k$$

$$2\pi^2 Km\left(\frac{1}{R_1} - \frac{1}{R_2}\right) = 2\pi^2 Km\frac{1}{R_1}$$

This equation will hold only when R_2 approaches infinity. So the planet will now ascend perpetually. Now, if planet 2 moves outward to the orbit of planet 3 and if during this outward motion the gravitational attracting power of the sun once again becomes one half of its actual value.

$$Gain\ in\ U = Loss\ in\ E_k$$

$$\Delta U = \ \Delta E_k$$

$$\frac{1}{2}4\pi^2 Km_2\left(\frac{1}{R_2} - \frac{1}{R_3}\right) = \frac{1}{2}m_2 V_2^2 - \frac{1}{2}m_2 V^2$$

Where V is the speed of planet 2 when it has reached the orbit normally occupied by planet 3.

$$\therefore 2\pi^2 K\left(\frac{1}{R_2} - \frac{1}{R_3}\right) = \frac{1}{2}V_2^2 - \frac{1}{2}V^2$$

$$= \frac{1}{2}\left(\frac{2\pi R_2}{T_2}\right)^2 - \frac{1}{2}V^2$$

$$= \frac{2\pi^2 R_2^2}{T_2^2}\frac{R_2}{R_2} - \frac{1}{2}V^2$$

$$= 2\pi^2\frac{K}{R_2} - \frac{1}{2}V^2$$

$$\therefore 2\pi^2 K\frac{1}{R_3} = \frac{1}{2}V^2$$

$$V^2 = 4\pi^2 K\frac{1}{R_3} = V_3^2$$

So, we conclude that if a planet moves away from the sun in a straight line with its normal orbital speed but if the sun's gravitational force is ½ of what it actually is then each planet reaches the orbit of any outer planet and it will have there a linear speed exactly equal to the orbital speed of the planet that occupies that orbit.

Thus, Newton concluded if all planets ascend at once and ascend in the same line, they will constantly in ascending become nearer and nearer together and their motion will constantly approach to an equality and become at length zero.

The converse of this statement will apply to Galileo-Plato concept:

When the planets reach zero motion position, let the motion of all these planets be reversed and let fall. Each planet would then arrive at its own orbit with its proper normal orbital speed. As the planets reach their respective orbits then, their motion turned sideways and at the same time the gravitational power of the sun doubled.

Newton said at this point that the Divine power is here, required in a double respect.

1. To turn the straight line motion of the falling planets into a side motion.
2. To double the attractive power of the sun at the same time.

If the attractive power of the sun is not doubled, then the sun will not be able to hold the planets and they will go into the highest heavens in parabolic lines.

The unity of Galileo's scientific life, combining observational astronomy and mathematical physics, comes from his dedication to a sun-centred universe, a dedication reinforced in some way by every major discovery he made in either physics or astronomy. Having been the instrument by which the glorious aspects of the creation in the heavens first had been fully revealed to a mortal, Galileo must have has a special sense of urgency to convert all his fellowmen to the true i.e. the Copernican system of the universe. His conflict with the Roman Catholic Church arose because deep in his heart, Galileo was a true believer. There was for him no path of compromise. No way to have separate secular and theological cosmologies. If the Copernican system was true, as he believed, then what else could Galileo do but fight with every weapon he had, logic, rhetoric, scientific observation, mathematical theory and cunning insight, to make his church accept a new system of the universe. We may catch a glimpse of the spirit of this great man, as I think of him, after his trial and condemnation, living under a kind of house arrest, completing his greatest scientific work, “Discourse and Demonstrations Concerning Two New Sciences.” This book was the base from which Newton began his great exploration of the dynamical principles of a sun-centred universe.

Bibliography

1. Basu & Chatterji; The Pre-University and Intermediate Physics.
2. Borowitz & Bornstein; A Contemporary View of Elementary Physics.
3. Cohen, Bernard J.; The Birth of a New Physics.
4. Cooper, Lane; Aristotle, Galileo and the Tower of Pisa.
5. Drake, Stillman; Discoveries and Opinions of Galileo.
6. Drake, Stillman; Galileo Galilei, A Biography and Inquiry into his Philosophy of Science.
7. McMullin, Ernan; Galileo, Man of Science.
8. Seeger, Raymond J.; Galileo Galilei, His Life and his Works.
9. Shortley and Williams; Elements of Physics.

Appendix B: Facsimile of the Original Manuscript (1972)

This appendix reproduces scanned images of the original manuscript pages of “Galileo” (1972), provided to preserve the historical form and provenance of the document. The fascimile can be found online at https://github.com/ishgosw/Galileo-as-Physicist-and-Polemicist